

**ISTC Project A-1512p**

**Physical Foundations of Plasma Microwave Sources Based  
on Anomalous Doppler Effect**

**Final Project Technical Report**

**on the work performed from June 01, 2007 to May 31, 2008**


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14. ABSTRACT  <p>This report results from a contract tasking Institute of Radiophysics and Electronics National Ac. Sci of Armenia as follows: The project is concerned microwave relativistic plasma electronics, dealing with problems of converting the energy of intense electron beams into high-power coherent electromagnetic radiation. This is one of the main topics of AFOSR's interests on basic research in physics and electronics outlined in the Broad Agency Announcement, 2004 and 2005. Plasma filling of microwave devices allows operating at higher beam currents and gradually changing frequency of output radiation. In addition plasma mitigates voltage depression and improves characteristics of output radiation. These and some other principal advantages, urge on following deep fundamental investigations on this topic for success in design of plasma devices that promise to be of record power and efficiency and of tunable frequency. Up to now theory of plasma-filled microwave devices is constructed for infinitely strong external longitudinal magnetic field. Authors of the theory are among the authors present project. They have great experience in physics of microwave devices on relativistic electron beam, as well as in physics of high current beam, plasma physics, and beam-plasma interaction. They published 11 books and 40 reviews on these topics. In a view of great scientific importance of the books, some of them are translated into English. Now, in the framework of present project, they intend to develop full theory of high power plasma-filled sources of microwave radiation in finite external magnetic field. Presently, theory of high power microwave plasma devices actually considers cylindrical waveguide with thin annular e-beam and coaxial thin annular plasma. External longitudinal magnetic field is assumed to be strong enough to freeze transversal motion of plasma and beam electrons. Dispersion properties of such systems make possible to control frequency of beam-plasma interaction (that is determined by intersection of straight line of Cherenkov resonance with dispersion curve) by changing plasma density. At changing over to higher frequencies dispersion properties of plasma-filled systems set upper limit on the value of external magnetic field. Thus, the problem must be solved with the regard of limiting value of external magnetic field. Theory of plasma-filled devices in finite external magnetic field is needed for following advance in design of high power plasma-filled sources. Besides, investigation of beam-plasma interaction in finite external magnetic field is important to develop theory of basic mechanisms of radiation of beam electrons: single particle and collective Cherenkov effects and anomalous Doppler Effect. Following primary problems are to be solved in the framework of present project: - Linear theory of beam-plasma interaction in finite magnetic field. - Nonlinear dynamics of beam-plasma interaction in finite magnetic field in uniform unbound and no uniform cross section system - Theory of beam plasma interaction in finite magnetic field with account of radiation output from system of finite length - Theory of beam plasma interaction in finite magnetic field with account of dissipation.</p>		

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## 1. Brief description of the Project and objectives

Plasma microwave generators on relativistic electron beams open up the best possibilities for direct conversion of beam kinetic energy to energy of electromagnetic radiation in large wavelength range [1]-[6]. Recent plasma devices actually correspond to cylindrical waveguide with thin annular e-beam and coaxial thin annular plasma in external longitudinal magnetic field that is strong enough to freeze transversal motion of plasma and beam electrons[1]-[3]. In these systems plasma oscillations are excited as a result of beam-plasma instability development. The waves are brought out via trumpet. Dispersion properties of such systems make possible to control output frequency (determined by intersection of straight line of Cherenkov resonance with the dispersion curve) and bandwidth by changing plasma density and geometry (distance between beam and plasma pipes). The advantages of plasma-filled systems were justified experimentally in the Institute of General Physics of Russian Ac. Sci and in other experiments

Theory of plasma-filled microwave devices in strong external magnetic field has been developed by authors of present project under guidance of their scientific leader A.A. Rukhadze. They published vast literature on plasma physics, beam-plasma interaction, and theory of microwave devices on relativistic electron beams (plasma filled as well as vacuum): 11 books and more than 40 reviews. Some of the books have been translated into English [1], [7]-[8].

At changing over to higher frequencies upper limitation on the value of external magnetic field arises. Thus, the problem must be solved with the regard of finite value of external magnetic field. Given project has been aimed to develop theory of plasma devices in finite external longitudinal magnetic field. Including investigation of the mechanisms of wave excitation, nonlinear dynamics of instability development in no-uniform cross section waveguide.

This theory is also important for optimization of plasma microwave oscillators and amplifiers (for reducing their prices). In addition, investigations of beam-plasma interaction in finite external magnetic field are important for development of theory of fundamental mechanisms of radiation in plasma such as collective and single particle Cherenkov effects and anomalous Doppler Effect.

In the framework of present project following first-priority problems related to the configuration of microwave sources in finite external field:

- Linear theory of beam-plasma interaction in finite magnetic field.
- Nonlinear dynamics of beam-plasma interaction in finite magnetic field in unbound uniform and no-uniform cross section systems.
- Theory of beam plasma interaction in finite magnetic field with account of radiation output from system of finite length.
- Theory of beam plasma interaction with account of dissipation in configuration of microwave sources in finite external magnetic field

## 2. Research methods

In the framework of present project the mechanisms of excitations of plasma oscillations by high current relativistic electron beam and their nonlinear dynamics in finite external magnetic field are theoretically investigated.

- Linear analysis has been carried out based on solution of dispersion equation. Electron beam and plasma are described by linearized set of hydrodynamic equations.
- Nonlinear dynamics of beam-plasma interaction in finite magnetic field has been investigated numerically based on large particle (particle in cell) method.
- For adequate account of the reflection in plasma microwave sources the reflective properties of the electrodynamic system has been determined. The analysis has been carried out based solutions of Helmholtz equation.
- For investigations of influence of dissipation on beam-plasma instabilities in configuration of microwave sources we used methods of plasma electrodynamics that are developed in books of the authors of this project (e.g. [1], **[Error! Reference source not found.]-[8]**).

## 3. Results

The basic mechanisms of beam-plasma (BP) instability that underlay on modern development of plasma microwave electronics are single-particle (Tomson-type) and collective (Raman-type) Cherenkov effects. If the beam density is low enough (as compared to plasma density) the conventional Cherenkov effect realizes under resonance  $\omega = k_z u$  ( $\omega$  is frequency,  $k_z$  - longitudinal wave number,  $u$  - the velocity of the beam electrons). As a rule, the value of external longitudinal magnetic field  $B_0$  in actual experiments is high enough and the Larmor frequency of electrons  $\Omega_e = eB_0/mc$  is higher than plasma frequency  $\omega_p = \sqrt{4\pi e^2 n_p / m}$ , ( $n_p$  is plasma density,  $e$  and  $m$  are the charge and the mass of electron). That is why most of theoretical investigations have been carried out in limit of infinite value of external magnetic field. At the same time new resonances arise in finite external magnetic field as well as new mechanisms of instability: normal and anomalous Doppler effects. These effects appear themselves in following conditions:  $\omega = k_z u \pm \Omega_e / \gamma$ , where  $\gamma = (1 - u^2/c^2)^{-1/2}$  - is relativistic factor of the beam electrons. In spite of the Cherenkov and the Doppler effects reveal themselves in different frequency and wave vector ranges, these effects compete and the competition can essentially influence on development of instability in the system.

The normal Doppler effect corresponds to wave non-transparency in BP system and doesn't lead to instability. But if anomalous Doppler effect takes place, the developing instability leads to increasing of transversal component of velocity of the beam electrons and to decreasing of the longitudinal component. As a result the velocity distribution of the beam electrons can spread out and the development of Cherenkov-type instability can be deranged.

We consider beam-plasma system consisting cylindrical waveguide of radius  $R$  with plasma and electron beam homogeneous along axis. In the cross-section the plasma and the beam correspond to thin-walled pipe with the mean radii  $r_{p,b}$  ( $r_p \neq r_b$ ) and thicknesses  $\Delta_{p,b}$ . The basic result of linear theory is the dispersion relation that takes into account the interaction of low frequency cable wave of plasma filled waveguide with the thin annular e-beam. It may be written in following form

$$\omega^2 - \Omega_p^2 = G_{dp} \frac{\omega_p^2 \omega_b^2 \gamma^{-1}}{(\omega - k_z u)^2 - (\Omega_e / \gamma)^2} + G_{ch} \frac{\omega_p^2 \omega_b^2 \gamma^{-3}}{(\omega - k_z u)^2}$$

where  $\omega_b$  is the Lengmuir frequency of the beam electrons, and expressions

$$\Omega_p^2 = \omega_p^2 \frac{\chi_0^2}{k_{\perp p}^2}, \quad \frac{1}{k_{\perp p}^2} = r_p \Delta_p I_0^2(\chi_0 r_p) \left[ \frac{K_0(\chi_0 r_p)}{I_0(\chi_0 r_p)} - \frac{K_0(\chi_0 R)}{I_0(\chi_0 R)} \right]$$

determine transversal structure of considered system, at that

$$\chi_0^2 = k_z^2 - \omega^2/c^2, \quad G_{dp} = \chi_0^2 \frac{r_b \Delta_b}{r_p \Delta_p k_{\perp p}^2} \frac{I_1^2(\chi_0 r_b)}{I_0^2(\chi_0 r_p)} \left( 1 - \frac{u^2}{c^2} \frac{\Omega_p}{k_z u} \right), \quad G_{ch} = \chi_0^2 \frac{r_b \Delta_b}{r_p \Delta_p k_{\perp p}^2} \frac{I_0^2(\chi_0 r_b)}{I_0^2(\chi_0 r_p)}.$$

(here  $I_{0,1}$  and  $K_0$  are the modified Bessel and MacDonald functions).

Based on numerical modeling we analyzed beam-plasma interaction. Approximate expressions for the growth rates of excited by the e-beam cable wave have a form

$$\omega = \Omega_p + \delta\omega,$$

$$\delta\omega = \begin{cases} \frac{-1 + i\sqrt{3}}{2} \left( \frac{1}{2} \frac{G_{ch} \omega_p^2 \omega_b^2}{\Omega_p \gamma^3} \right)^{\frac{1}{3}} & \text{-- Cherenkov Effect,} \\ \frac{1}{2} i \omega_{\text{Don.}} \left( \frac{G_{dp} \omega_p^2 \omega_b^2}{\Omega_p \Omega_e} \right)^{\frac{1}{2}} & \text{-- anomalous Doppler Effect.} \end{cases}$$

If the growth rate of the instability due to anomalous Doppler effect is higher than the growth rate of Cherenkov-type instability the Cherenkov-type instability does not develop at all. One can easily see this is possible only in comparatively low external magnetic fields. But even if this condition does not hold, and the growth rate of Cherenkov-type instability is greater, that all the same: development of the instability due to anomalous Doppler effect takes place. It develops in other (shorter) wavelength range and development of long wavelength instability can not suppress it. That is why the development of slower instability of anomalous Doppler type can lead to full derangement of the Cherenkov-type instability on late stages of its development.

In the framework of the project a set of equations is obtained that uniformly describes development of following types of instabilities in beam-plasma systems in finite external magnetic field: Cherenkov-type instability that develops on plasma branch of oscillations; Cherenkov-type instability that develops on cyclotron branch of plasma oscillations and anomalous Doppler type instability on the plasma and cyclotron branches of oscillation. System consisting of cold plasma and straight-line electron beam was considered. Consideration has been carried out in potential approximation.

Let  $z$  is the coordinate along unperturbed motion of the beam and the external magnetic field,  $x$  is transversal coordinate and there is no dependence on the third coordinate. The expression for the beam and the plasma densities may be written as

$$\rho_\alpha(t, x, z) = en_{0\alpha} \iint \delta(x - x_\alpha) \delta(z - z_\alpha) dx_0 dz_0 \quad (1)$$

where  $\alpha = p, b$  for the plasma and the beam respectively,  $x_\alpha(t, x_0, z_0)$  and  $z_\alpha(t, x_0, z_0)$  are the trajectory of the particle that begins the motion from the point  $x_0, z_0$  at the time  $t = 0$ .

Represent the potential of the electric field in the form

$$\frac{e}{m} \varphi(t, x, z) = \frac{1}{2} \sum_{n,s=0} \left( \varphi_{ns}(t) \exp(isk_{\perp} x) \exp(ink_{\parallel} z) + \text{c.c.} \right) \quad (2)$$

where  $e$  and  $m$  is the charge and the mass of electron,  $k_{\parallel}$  and  $k_{\perp}$  are the basic values of longitudinal and transversal wavenumbers. These values are determined particularly by the structure of initial perturbation thrown into considered system. Substituting the (1) and (2) into the Poisson equation we have



$$\varphi_{ns}(t) = q_{ns}^{-2} \sum_{\alpha=p,b} \omega_{\alpha}^2 \rho_{\alpha ns}(t) \quad (3)$$

Here where  $\omega_{\alpha} = \sqrt{4\pi e^2 n_{0\alpha}/m}$  are Longmuir frequencies of  $\alpha$  - type particles and

$$\rho_{\alpha ns}(t) = \frac{k_{\perp}}{2\pi} \frac{k_{\parallel}}{\pi} \int_0^{2\pi/k_{\perp}} dx_0 \int_0^{2\pi/k_{\parallel}} dz_0 \exp(-isk_{\perp}x_{\alpha}(t, x_0, z_0)) \exp(-ink_{\parallel}z_{\alpha}(t, x_0, z_0)) \quad (4)$$

are dimensionless amplitudes of the density perturbations of  $\alpha$  - type particles (dimensional amplitudes may be obtained by multiplication of (4) by unperturbed densities  $n_{0\alpha}$ )  
 $q_{ns}^2 = s^2 k_{\perp}^2 + n^2 k_{\parallel}^2$ .

As a result the potential of electric field takes following form

$$\frac{e}{m} \varphi(t, x, z) = \frac{1}{2} \sum_{\alpha} \omega_{\alpha}^2 \left( \sum_{n,s=0} q_{ns}^{-2} \rho_{\alpha ns}(t) \exp(isk_{\perp}x) \exp(ink_{\parallel}z) + \text{c.c.} \right) \quad (5)$$

Using (5), equations of motion of the plasma ( $\alpha = p$ ) and the beam ( $\alpha = b$ ) electrons may be written as

$$\frac{d^2 z_p}{dt^2} = -\frac{1}{2} ik_{\parallel} \sum_{\alpha=p,b} \omega_{\alpha}^2 \left( \sum_{n,s=0} n q_{ns}^{-2} \rho_{\alpha ns}(t) \exp(isk_{\perp}x_p) \exp(ink_{\parallel}z_p) - \text{c.c.} \right), \quad (6)$$

$$\frac{d^2 x_p}{dt^2} + \Omega_e^2 (x_p - x_0) = -\frac{1}{2} ik_{\perp} \sum_{\alpha=p,b} \omega_{\alpha}^2 \left( \sum_{n,s=0} s q_{ns}^{-2} \rho_{\alpha ns}(t) \exp(isk_{\perp}x_p) \exp(ink_{\parallel}z_p) - \text{c.c.} \right),$$

$$\frac{d^2 z_b}{dt^2} = -\frac{1}{2} ik_{\parallel} \sum_{\alpha=p,b} \omega_{\alpha}^2 \left( \sum_{n,s=0} n q_{ns}^{-2} \rho_{\alpha ns}(t) \exp(isk_{\perp}x_b) \exp(ink_{\parallel}z_b) - \text{c.c.} \right), \quad (7)$$

$$\frac{d^2 x_b}{dt^2} + \Omega_e^2 (x_b - x_0) = -\frac{1}{2} ik_{\perp} \sum_{\alpha=p,b} \omega_{\alpha}^2 \left( \sum_{n,s=0} s q_{ns}^{-2} \rho_{\alpha ns}(t) \exp(isk_{\perp}x_b) \exp(ink_{\parallel}z_b) - \text{c.c.} \right).$$

In linear approximation the following dispersion relation results from (6) and (7)

$$k_{\parallel}^2 \left( 1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega_b^2}{(\omega - k_{\parallel}u)^2} \right) + k_{\perp}^2 \left( 1 - \frac{\omega_p^2}{\omega^2 - \Omega_e^2} - \frac{\omega_b^2}{(\omega - k_{\parallel}u)^2 - \Omega_e^2} \right) = 0 \quad (8)$$

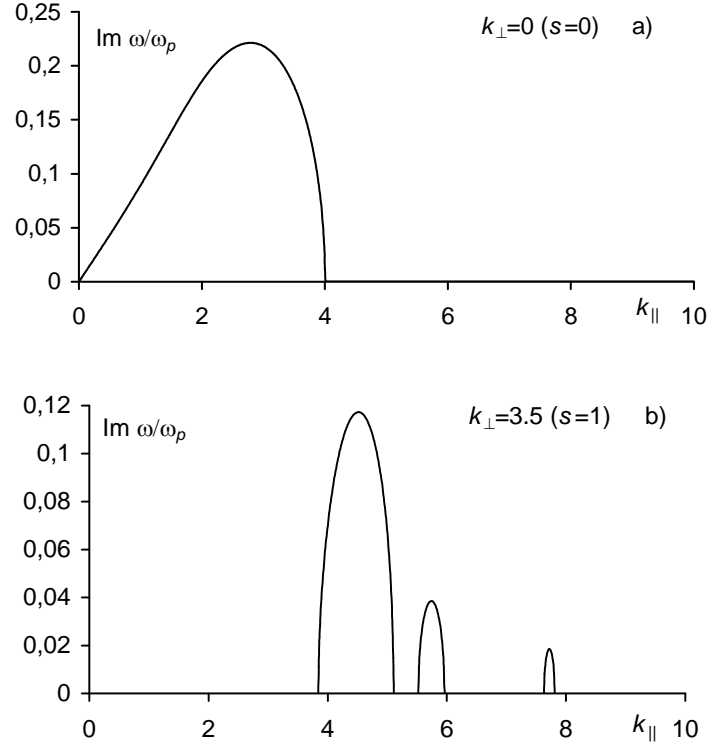
Numerical analysis of (8) has been carried out for following values of the parameters

$$\omega_b^2 / \omega_p^2 = 0.05, \quad \Omega_e / \omega_p = 1.5, \quad k_{\perp}u / \omega_p = 1.4, \quad k_{\perp} = 3.5 cM^{-1}$$

Fig 1 represents the dependences of the growth rate of the beam-plasma instability in finite external magnetic field on longitudinal wavenumber  $k_{\parallel}$  for various transverse modes  $s=0$ ,  $s=1$ . Fig 1a corresponds to  $s=0$ . In this case there is one region of instability only. It is related to stimulated Cherenkov radiation of the low frequency plasma wave. The dispersion relation (8) shows that there is no other instabilities. Recall that in the equation (8)  $k_{\perp}$  actually means  $sk_{\perp}$ . The Cherenkov type instability under mode  $s=0$  is due to stimulated Cherenkov effect ( $\text{Im } \omega > \omega_b$ ).

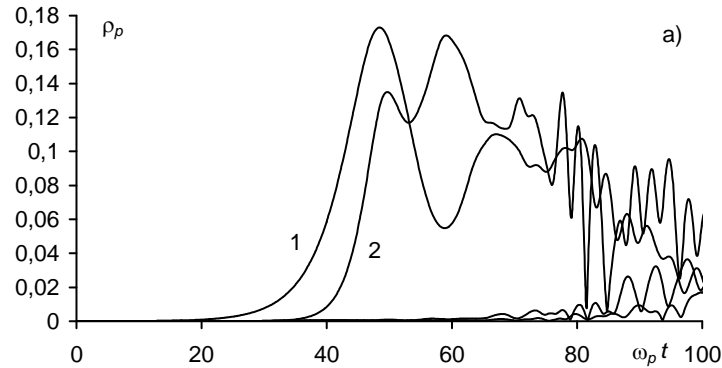
Fig 1b presents the growth rates of the instabilities under  $s=1$ . There are three regions that do not overlap. The left-hand region, where the growth rate is maximal, is related to stimulated Cherenkov radiation of high frequency cyclotron wave in plasma. This instability is due to stimulated Collective Cherenkov effect ( $\text{Im } \omega < \omega_b$ ). Following two instability region (to the sideways of large  $k_{\parallel}$ ) are related to radiation of low frequency plasma wave and high frequency cyclotron wave in conditions of anomalous Doppler effect. The growth rates under anomalous Doppler effect are essentially lower than the growth rates of Cherenkov-type instabilities. Note, there is no instability on transversal mode  $s=1$  that is due to Cherenkov radiation of low

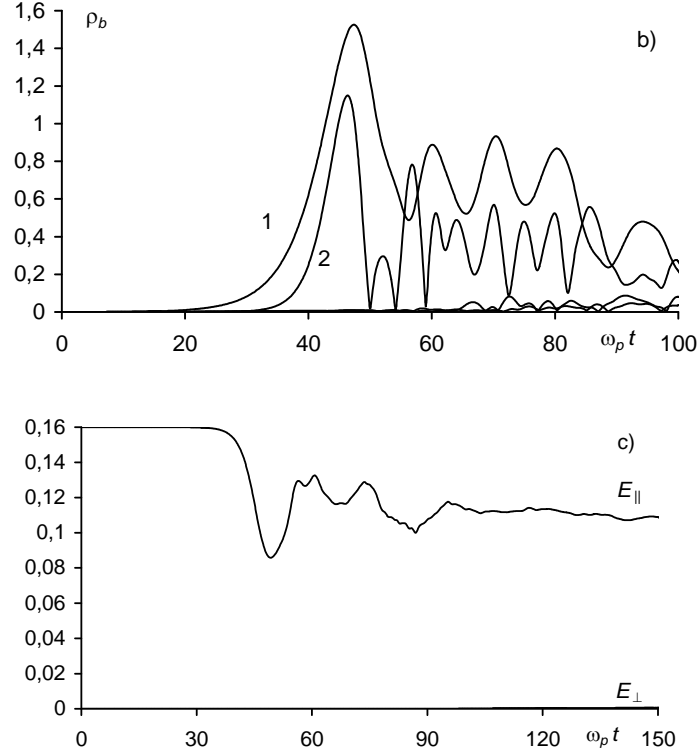
frequency plasma wave, because the needed condition for this instability  $\omega_p > k_{\perp} u$  does not satisfy for given value of  $k_{\perp}$ .



**Fig. 1.** The dependences of the growth rates of the instability  $\text{Im } \omega / \omega_p$  vs longitudinal wavenumber  $k_{||}$  for various values of  $k_{\perp}$ .

Now begin to analyze the nonlinear dynamics of the instability development. The dependencies of the beam and the plasma densities on the time are presented on the Figs 2a,b. The dependencies are calculated from the expression (4). On the linear stage the harmonics  $s = 0$ ,  $n = 1, 2$  are excited (see curves 1,2). The first one corresponds to Cherenkov excitation of the low frequency plasma wave. There takes place energy transfer to higher harmonics due to nonlinear interaction of various waves. Under  $s \neq 0$  the growth rates are less and the excitation of these waves is weaker. The changes of the energy of longitudinal and transversal motion are presented on the fig 2c. This fig shows that the transversal motion actually does not





**Fig. 2.** The dependences of the densities of the plasma (a) and the beam (b) electrons, and the dynamics of changes of longitudinal  $E_{||}$  and transversal  $E_{\perp}$  energy of the beam (c).

excite, and the instability is of Cherenkov-type. This is related to relatively low growth rate of anomalous Doppler-type instability and it has no time to develop. Insignificant increase of the transversal energy is caused by the motion of the electrons be subjected to transversal component of the electric field.

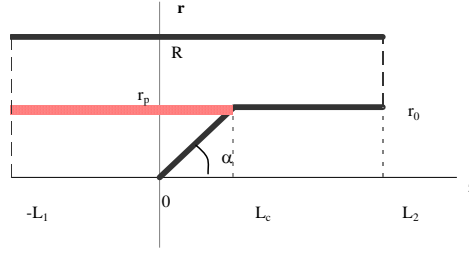
In the framework of the project the problem of definition of reflective properties of the longitudinally confined system on the boundary between beam-plasma system and the emitting trumpet was considered.

For modeling of the reflective properties we have considered (see Fig 3) part of waveguide of radius  $R$ :  $-L_1 < z < L_2$ . The part  $0 < z < L_c$  is occupied by metallic cone. The angle on its vertex is  $2\alpha$ . On  $z = L_c$  this cone turns into metallic coaxial, inner radius of which is  $r_0$ . Plasma layer with radial profile described by expression

$$\omega_p^2(r) = \omega_p^2 \delta_p \delta(r - r_p)$$

( $\omega_p$  is the Longmuir frequency of the plasma,  $\delta_p$  is the thickness of the layer,  $r_p$  is the mean radius,  $\delta(x)$  is the Dirac function) is placed on the left-hand side of the cone. The plasma wave, if it propagates along  $z$  axis in positive direction, reflects partially from the conical region. As a result of this partial reflection backward waves arise. Propagation of the wave in the region  $z > L_c$  leads to creation of proper waves of the waveguide with inner radius  $r_0$ , and outward radius  $R$ .

Calculation of the reflection coefficient was carried out by solving the Maxwell's and hydrodynamic equations. We considered scattering of wave train with fixed (in the framework of given calculation) carrier frequency, but the wave vector  $k_z$  may be obtained from the



**Fig. 3.** *Metallic resonator with thin plasma and emitting trumpet.*

respective dispersion relation. Thus by varying the frequency it is possible to obtain the dependence of the reflection coefficient on the frequency – its dispersion.

For modeling of the reflection processes in abovementioned system we use linearized hydrodynamic equations for describing the current density in plasma.

$$\frac{\partial j_p}{\partial t} = \frac{\omega_p^2}{4\pi} E_z(r_p)$$

where  $E_z$  is the longitudinal electric field on the plasma layer.

In the vacuum region the set of Maxwell's equations are

$$\frac{\partial E_r}{\partial t} = -c \frac{\partial B_\phi}{\partial z}, \quad \frac{\partial E_z}{\partial t} = -\frac{c}{r} \frac{\partial(rB_\phi)}{\partial r}, \quad \frac{\partial B_\phi}{\partial t} = c \left( \frac{\partial E_z}{\partial r} - \frac{\partial E_r}{\partial z} \right).$$

With account of infinite conductivity of the waveguide's walls, one can use following boundary conditions for the electric field:

$$E_z(r=R)=0, \quad E_z(r=r_0, z>L_c)=0, \quad E_\tau(0<z<L_c)=0,$$

where  $E_\tau$  is the component of the electric field, tangential to conic surface of the trumpet.

Account of the symmetry gives additional conditions for the field on the axis:

$$E_r(r=0)=0, \quad B_\phi(r=0)=0.$$

On the plasma the boundary conditions have following form

$$E_r(r_p+0)-E_r(r_p-0)=4\pi\sigma_p, \quad B_\phi(r_p+0)-B_\phi(r_p-0)=\frac{4\pi}{c}j_p,$$

where  $\sigma_p(z,t)$  and  $j_p(z,t)$  are the surface charge and current densities in plasma that are coupled by continuity equation

$$\frac{\partial \sigma_p}{\partial t} + \frac{\partial j_p}{\partial z} = 0$$

Under  $r=r_p$  the component  $E_z$  is continuous but its derivative undergoes discontinuity

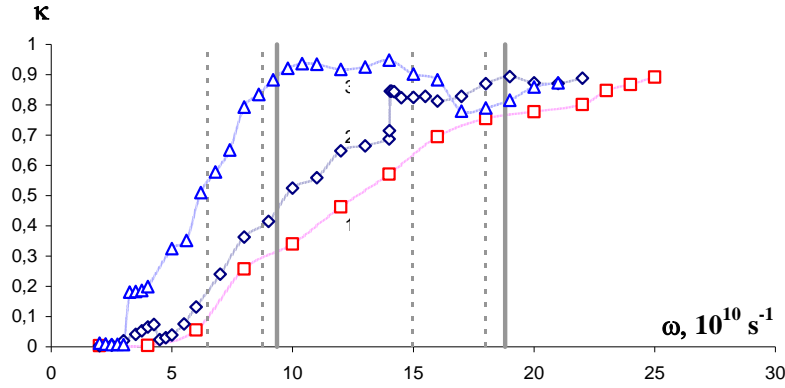
$$\frac{\partial E_z(r_p+0)}{\partial r} - \frac{\partial E_z(r_p-0)}{\partial r} = \frac{4\pi}{c^2} \frac{\partial j_p}{\partial t} + 4\pi \frac{\partial \sigma_p}{\partial z}$$

On the plane  $z=-L_1$  no stationary boundary condition is given that is responsible for excited plasma wave

$$E_r(t, z=-L_1) = \Psi(r) \begin{cases} 0, & t < 0 \\ P(t) \sin \omega t, & 0 < t < t_0 \\ 0, & t > t_0 \end{cases}$$

where  $t_0$  is the impulse duration,  $P(t)$  is the impulse envelope,  $\Psi(r)$  is the proper function of excited plasma wave.

We defined the coefficient of reflection as a ratio of energy flux of reflected wave to that of incident wave. The frequency dependencies of the reflection coefficients for various angles  $\alpha$  of the cone are presented in the Fig 4. The curve 1 corresponds to  $\alpha = 90^\circ$ , the curve 2 – to  $\alpha = 45^\circ$ , the curve 3 – to  $\alpha = 10^\circ$ . The cutoff frequencies of volume waves in the region  $z < 0$  are denoted by dotted upright lines but in the region  $z > L_c$  by continuous upright lines. It is seen with increase in frequency of excited wave the reflection coefficient increases also and tends to unity. In the region of intermediate frequencies the values of the coefficient are of order 0.3 – 0.7. The case  $\alpha = 90^\circ$  provides the best matching i.e. the sharp transition of the beam-plasma waveguide to emitting trumpet.



**Fig. 4.** The frequency dependencies of the reflection coefficients for various angles  $\alpha$  of the vertex of emitting trumpet (1 -  $\alpha = 90^\circ$ , 2 -  $\alpha = 45^\circ$ , 3 -  $\alpha = 10^\circ$ ).

In the framework of present project the effect of dissipation in configuration of plasma microwave devices has also been investigated. In the configuration with spatially separated beam and plasma proper oscillations of the beam play an important role. Of special importance is the excitation of the negative energy beam wave (NEBW). Its growth leads to instability due to sign of energy. Characteristic peculiarity of this instability (as compared to conventional beam-plasma instability) is its growth rate attains maximum under collective Cherenkov resonance. Dissipation in the system also leads to the growth of the same wave. In these conditions the role of dissipation increases, as it intensifies the growth of the NEBW. Actually, the configuration creates a superposition of two factors both of which lead to excitation of NEBW. With increase in dissipation level this superposition leads to dissipative beam instability (DBI) of new type. Its growth rate has more critical dependence on dissipation (as compared to conventional DBI).

One of the basic trend of microwave electronics – to increase output frequency – leads to decreasing of the skin depth in the walls of resonators. Their quality factor  $Q$  decreases and actually dissipation increases. One more reason of the increase of the dissipation should be mentioned. In waveguide systems return current flows mainly via metallic surfaces. With increase in the beam current the return current increases also. With account of decreasing of the skin depth and finite conductivity of metallic surfaces this significantly increases the level of dissipation in the system. All this creates favorable conditions for developing of the new type of DBI.

Exact solution of the problem justifies this reasoning. The influence of dissipation in abovementioned configuration has been investigated quantitatively. Cylindrical waveguide filled by plasma and penetrated by electron beam has been considered. The beam and the plasma are thin annular with mean radii  $r_p$  and  $r_b$ ;  $\delta_p$  and  $\delta_b$  are their thicknesses. We have made following expedient for theoretical model assumption: the plasma and beam are not just thin but infinitesimal thin. This assumption allows using approximate boundary conditions carried out in [9] that essentially simplify consideration. Dissipation has been taken into account by introducing collision frequency  $\nu$  in plasma. Obtained dispersion relation describes interaction of the electron beam with low frequency plasma wave. It may be written as follows (for symmetric modes)

$$\left\{ \omega^2 - \delta_p r_p \kappa^2 \omega_p^2 \left( 1 - i \frac{\nu}{\omega} \right) G_p \right\} \left\{ (\omega - ku)^2 - \delta_b r_b \kappa^2 \frac{\omega_b^2}{\gamma^3} G_b \right\} = \delta_p r_p \delta_b r_b \omega_p^2 \frac{\omega_b^2}{\gamma^3} \kappa^4 \frac{I_0^2(\kappa r_b)}{I_0^2(\kappa r_p)} G_p^2 \quad (*)$$

where  $\omega$  is the perturbations frequency  $k$  is the wave vector along axis,  $\kappa^2 = k^2 - \omega^2 / c^2$ ,  $\omega_{p,b}$  - are the Longmuir frequencies of the plasma and the beam  $\gamma = (1 - u^2 / c^2)^{-1}$ ,  $u$  is the velocity of the beam electrons,  $G_{p,b}$  are the geometric factors of the plasma and the beam

$$G_{p,b} = I_0^2(\kappa r_{p,b}) \left[ \frac{K_0(\kappa r_{p,b})}{I_0(\kappa r_{p,b})} - \frac{K_0(\kappa R)}{I_0(\kappa R)} \right]$$

The dispersion relation indeed leads to the new type of DBI. Its growth rate is given by

$$(\text{Im } \omega)_{\text{und}}^{(\nu)} = \frac{(ku)^2}{\nu} \frac{G}{2\gamma} \sqrt{\alpha} \quad (**)$$

where  $\alpha$  is the parameter that characterizes beam current value. It is equal to the ratio of the beam current to limiting vacuum current. The expression for the growth rate is written for underlimiting e-beams i.e.  $\alpha < 1$ . The parameter  $G$  shows level of beam-plasma coupling i.e. the overlap of the beam and the plasma fields. It complicatedly depends on the parameters. (\*\*) is obtained under weak coupling  $G \ll 1$  and depends on the dissipation inverse proportionally.

As compared to conventional DBI the dependence became more critical, i.e.  $\nu^{1/2} \rightarrow \nu^{-1}$ .

The expression (\*\*) makes quotation, for simplicity, in limit of strong external field. Explicit expression for the growth rate under arbitrary external field is not quoted here. The equations for the plasma and the beam wave (equality to zero of first and second braces in left-hand side of (\*)) have no analytical solution. The results are obtained numerically. But the structure of the equation (\*) shows that the growth rate inevitably depends on dissipation inverse proportionally.

Thus abovementioned configuration of microwave devices favors development of the new type of DBI with inverse proportional dependence on dissipation. Its properties and development conditions show that it can adversely affect on the operation in conditions of collective Cherenkov effect in short wavelength limit. This should be taken into account upon design of future devices.

The transient processes in microwave devices are determined by rate of growth of initial perturbation. The influence of dissipation on the development dynamics of the initial perturbation in the same configuration is also investigated in the framework of the project. The solution of the problem was carried out based on method developed in [10]. This method has been modified according to conditions of collective Cherenkov effect. The results show that dissipation significantly effects on growth rates and suppresses low-velocity (lower than the beam velocity) modes. In the limit of high-level dissipation, unstable perturbations move at beam velocity. I.e. if one considers injection of an e-beam into plasma-filled waveguide and its further propagation, the DBI develops mainly near beam front.

## 4. Conclusion

1. Based on numerical modeling we analyzed beam-plasma interaction. Increments values of beam-plasma system in finite magnetic field are obtained. The analysis leads to following conclusions:

- Anomalous Doppler effect can essentially influence on the character of development of Cherenkov type instability only in presence of magnetic field of moderate value, when Larmor frequency of the electrons is of order plasma frequency.
- The development of the instability of anomalous Doppler type leads to increasing of transversal component of the velocity of beam electrons. In its turn this can leads to full derangement of the Cherenkov type instability.

2. A set of equations is obtained that uniformly describes development of following types of instabilities in beam-plasma systems in finite external magnetic field: Cherenkov-type instability that develops on plasma branch of oscillations; Cherenkov-type instability that develops on cyclotron branch of plasma oscillations and anomalous Doppler type instability on the plasma and cyclotron branches of oscillation. A system consisting of cold plasma and straightline electron beam was considered. Consideration has been carried out in potential approximation. Greater growth rates of Cherenkov-type instabilities make namely this mechanism of wave excitation dominant. However, in transversally bounded systems the instabilities of this type have a threshold related to plasma density and in transversally bounded systems basic mechanism of wave excitation can that anomalous Doppler type.

3. The problem of definition of reflective properties of the longitudinally confined system on the boundary between beam-plasma system and the emitting trumpet was considered. The dependencies of the reflection coefficient are obtained. There is strong frequency dispersion of the reflection coefficient. For wide range of the frequencies the values of the reflection coefficient fall in the range 0.3 – 0.7. The best matching of the beam-plasma system with the emitting trumpet is provided by the cone angle  $90^\circ$ .

4. In configuration of plasma microwave devices in finite external magnetic field new type of dissipative beam instability (BI) can develop. Properties of this instability and conditions of its development show that it reveals itself as additional factor that should be taken into account under design of microwave devices (especially under operation in regime of collective Cherenkov effect, in short wavelength range).

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Attachment 1: (p.14)

Attachment 2: (p15-16)

Attachment 3:

**None**

List of published papers and reports with abstracts

List of presentations at conferences and meetings with abstracts

Information on patents and copy rights (List and describe patents and copyrights which were obtained or may be obtained as a result of the project)



**List of published papers and reports with abstracts**

1. I.N.Kartashov, M.V.Kuzelev, A.A.Rukhadze. *Plasma-Beam Electromagnetic Interactions in Finite Magnetic Field*. Plasma Physics Reports. 2008. (to be published)

**Abstract.** The excitation of oscillations of cylindrical plasma waveguide with thin annular plasma penetrated by thin annular e-beam is considered in linear approximation. The growth rates of the instabilities and the coefficients of spatial amplification of the beam-plasma system are obtained in regimes of Cherenkov and anomalous Doppler effects. A comparison with the transversally uniform system is carried out. The competition of various mechanisms of the instability is analyzed.

2. A.F.Aleksandrov, M.V.Kuzelev, A.A.Rukhadze. *Electromagnetic Waves in Cylindrical Plasma-Dielectric Systems*. Journal of Communications Technology and Electronics. 2008 (to be published).

**Abstract.** The present article is devoted to the statement of the theory of electromagnetic waves in cylindrical plasma and plasma-dielectric filling systems (waveguides). The case of isotropic cold electron plasma with sharp cross-section borders plasma-vacuum, plasma-dielectric, plasma-plasma and plasma-metal is considered. The special attention is given to surface plasma waves and their interactions with volume electromagnetic modes.

### List of presentations at conferences and meetings with abstracts

1. E. V. Rostomyan. *Dissipative instability under weak beam-plasma coupling*. Presentation on PPPS-07 (IEEE International Conference on Pulsed Power and Plasma Science, June 17 – 22, 2007, Albuquerque, New Mexico USA).

*Interaction of monoenergetic electron beam and plasma in a waveguide is considered in general form without specifying the cross-section of the waveguide. The influence of dissipation leads to the new type of dissipative beam instability with growth rate that has more critical (as compared to conventional) dependence on dissipation.*

2. E. V. Rostomyan. *Influence of Dissipation on Instability of Overlimiting Electron Beam*. Presentation on PPPS-07 (IEEE International Conference on Pulsed Power and Plasma Science, June 17 – 22, 2007, Albuquerque, New Mexico USA).

*Investigation considers interaction of overlimiting e-beam with plasma in presence of dissipation. New type of dissipative beam instability develops. Its growth rate has inverse proportional dependence on dissipation*

3. E. V. Rostomyan. *Dissipative Instability of Negative Energy Wave of an Intense Electron Beam* was accepted for presentation at ICPIG-XXVIII (Int. Conf. on Phenomena in Ionized Gases, July 15-20, 2007, Prague, Czech Republic).

*The influence of dissipation on excitation of negative energy beam wave is considered. High level of dissipation leads to a dissipative beam instability that differs from conventional.*

4. I.N.Kartashov, M.V.Kuzelev, A.A.Rukhadze. *Instabilities of Straight Electron Beam in Plasma in Finite Magnetic Field*. Presentation in Conference on Plasma Physics Zvenigorod, Russia.

*One-particle and collective Cherenkov effects are the basic mechanisms of the stimulated radiation of electromagnetic waves by a straight electron beam in plasma-filled systems in strong magnetic field. In case of finite external magnetic field, in addition to conventional Cherenkov instability on the plasma branch of oscillations, Cherenkov instability on the cyclotron branch arises also, as well as the instabilities of anomalous Doppler-type on low-frequency plasma and high-frequency cyclotron branches. Nonlinear dynamics of beam instability in plasma in a finite magnetic field is investigated by numerical simulation in conditions of joint influence of Cherenkov and anomalous Doppler effects.*

5. E. V. Rostomyan. *Influence of Dissipation on Beam-Plasma Interaction in Finite External Magnetic Field* is accepted for presentation at BEAMS'08 (17<sup>th</sup> Int Conf on High Power Particle Beams, July 6-11, Xi'an, China)

*Investigation considers beam-plasma interaction in waveguide in finite external longitudinal field in presence of dissipation. It is shown that the new type of dissipative beam instability discovered under strong external magnetic field, develops in finite external magnetic field also. It can essentially influence on the operation of microwave devices.*

6. E. V. Rostomyan. *Beam-Plasma Interaction in Presence of Dissipation in Finite External Magnetic Field*. Is accepted for presentation at ICPP-08 (International Congress on Plasma Physics 2008, September, 8-12, Fukuoka, Japan).

*Present investigation presented a new type dissipative beam instability, the maximal growth rate of which has inverse proportional dependence on dissipation. It develops in a waveguide with spatially separated beam and plasma in finite external longitudinal field*